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Cash-in-advance constraints in production and the functional distribution of income

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Abstract

This paper is an attempt to demonstrate the effects of cash-in-advance constraints on competitive outcomes. We study a simple dynamic economy with labor being the only factor of production and assume that the labor market opens before the goods market. We characterize the stationary monetary competitive equilibria (SMCE) of the financially constrained economy and show that none of the equilibria coincides with the Arrow-Debreu equilibrium since the real wages turn out to be always below the marginal product of labor in SMCE. We also study the neutrality of money and establish the conditions under which an unexpected jump in the money supply may be non-neutral.

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1. Introduction

In economies with complete commodity markets, if the production techniques exhibit constant returns to scale, competition is expected to wipe out pure profits of firms and hence to yield an outcome that is both favourable for workers and Pareto efficient. In such economies there is no role for money, let it be fiat or commodity.

However, the use of fiat money as a medium of exchange is inevitable if the agents are infinitely-lived, there is no alternative storable commodity and if no purchase is possible without currency. Macroeconomic models of fiat money with cash-in-advance constraints were proposed by Clower (1967) and were popularized especially with the papers by Lucas (1980, 1984, 1990) and Stokey and Lucas (1983, 1987). In these papers, consumers face cash-in-advance constraints in their purchases of a subset of commodities or assets; but firms are able to produce and sell their supply freely, without any financial constraints, so that the classical result of zero producers' profits follows under constant returns to scale technologies. Fuerst (1992) deviates from the above literature by imposing cash-in-advance constraints in all market transactions. This leads to the result that real wages fall below the marginal product of labor in equilibrium. Fuerst (1992) worked in the representative family framework of Lucas (1990) in order to concentrate on the business cycle implications of cash constraints in production.

In this paper, we focus on the issue of functional distribution of income. To that end, we study a deterministic model with heterogeneity across agents with respect to their factor and money endowments, objective functions and production technologies. We posit cash-in-advance constraints in all markets and assume that the labor market opens before the goods market. This form of sequencing in transactions has an important effect on equilibrium prices as the finance constraints limit the effective factor demands of firms which in return gives rise to lower real wages in equilibrium than in the Arrow-Debreu equilibrium where competition is perfect. We also find that in a special case where producers extract all of the social surplus, there exists a continuum of equilibria which can be Pareto ranked. In this case, we show that the level of money stock is not neutral if the nominal wages are exogeneously determined and are sticky.

Section 2 introduces the model and Section 3 characterizes the set of equilibria for various parameter values on technology and preferences. The final section gathers concluding remarks.

2. The model

We consider an economy involving two commodities at each time t, labor L_t and a nonstorable consumption good (apple) q_t. There are two types of agents indexed by $i=1,2$. Neither of the types values leisure, and the preferences of both types over the lifetime consumption are of the same form as given by $\Sigma_{t=0}^{\infty} \beta^t U(c_{it})$, where c_{it} is the consumption of agent i at time t, $U(.)$ is the common utility function for both agents showing the instantaneous satisfaction derived from consuming apples at each time, and $\beta \in (0,1)$ is the common discount factor. We assume $U'(.) > 0$ and $U''(.) < 0$.

Agent 1 has a labor endowment $\bar{L}$ and a constant returns technology $f_1(L)=L$ to convert labor into apples with marginal product of labor equal to one. Agent 2 has no labor endowment, but a better technology $f_2(L) = \gamma L$ which is also constant returns to scale with marginal product of labor greater than one ($\gamma>1$). Other than these production possibilities, there are no apple endowments.

We assume that wage contracts cannot be written on apples, and therefore we introduce money together with a cash-in-advance constraints in both labor and apple markets. Let $M_{i,t}$ denote the money holding of agent i at time t. We assume that money is perfectly storable and initially all the money, M, in the economy is owned by agent 2, that is, $M_{i,0}= 0$ and $M_{2,0}= M$. Now let w_t and p_t, respectively, denote the money wage rate and the price of apples at time t.

The timing of market transactions is as follows: Agent i starts period t with money holding $M_{i,t}$. First the labor market opens where labor is sold at price w_t. Then apple production takes place with the purchased labor. After the harvest of apples, goods market opens where apple is sold at price p_t.

Given the endowment structure described above, and the sequence of prices $\langle w_t, p_t \rangle$, we can write the infinite-horizon utility maximization problems of the two agents as follows:

Agent 1

(P1) max $\Sigma_{t=0}^{\infty} \beta^t U(c_{it})$

subject to for all t,

$c_{1,t} = (\bar{L}-L_t^s) + q_{1t}^d$,

$L^s_t \leq \bar{L}$,
$M_{1,t+1} = M_{1,t} + w_t L^s_t - p_t q^d_t$,
$M_{1,t}, c_{1,t}, q^d_t, L^s_t \geq 0$, and
$M_{1,t} = 0$ is given.

Agent 2
(P1) max $\Sigma_{t=0}^{\infty} \beta^t U(c_{2t})$
subject to for all t,
$c_{2,t} = \gamma L^d_t + q^s_t$,
$w_t L^d_t \leq M_{2,t}$
$M_{2,t+1} = M_{2,t} + w_t L^d_t - p_t q^s_t$,
$M_{2,t}, c_{2,t}, q^s_t, L^d_t \geq 0$, and
$M_{2,0} = M$ is given.

Note that the superscript "d" stands for demand and superscript "s" stands for supply. The non-negativity requirement for M_{1t} together with the third constraint in (P1) is in effect a cash-in-advance constraint for agent 1 on apple purchases. For agent 2, the cash-in-advance constraint is present in its labor market transactions and seen as the second constraint in (P2).

We assume that there is a large and equal number from each type of agents and hence be concerned with *monetary competitive equilibrium (MCE)*. Under the assumptions that initially all the currency is owned by agent 2 (the entrepreneur) and that there is no injection or withdrawal of money by the government, MCE consists of a sequence of apple prices, money wages, labor demands and supplies, apple demands and supplies and money holdings by the two agents such that at each date demands, supplies and money holdings are optimal under the given price and wage sequences, demand equals supply in both labor and apple markets, and money holdings sum up to the total (initial) money supply at each time.

Formally, we say that the sequence $\langle p_t, w_t, L^d_t, L^s_t, q^d_t, q^s_t, M_{1,t}, M_{2,t} \rangle$ is a MCE if,

(i) The sequence $\langle L^s_t, q^d_t, M_{1,t} \rangle$ solves (P1) and the sequence $\langle L^d_t, q^s_t, M_{2,t} \rangle$ solves

(P2) under $\langle w_t, p_t \rangle$,

(ii) $L^d_t = L^s_t$ and $q^d_{1t} = q^s_{1t}$, for each t,

(iii) $M_{1,t} + M_{2,t} = M$, for each t.

In particular we also define the *stationary monetary competitive equilibrium (SMCE)*, where the prices, wages, demands, supplies, money holdings are all constant over time in a way that is consistent with dynamic optimization and market clearing. In the rest of the paper we will try to characterize the set of SMCE.

3. Characterization of SMCE

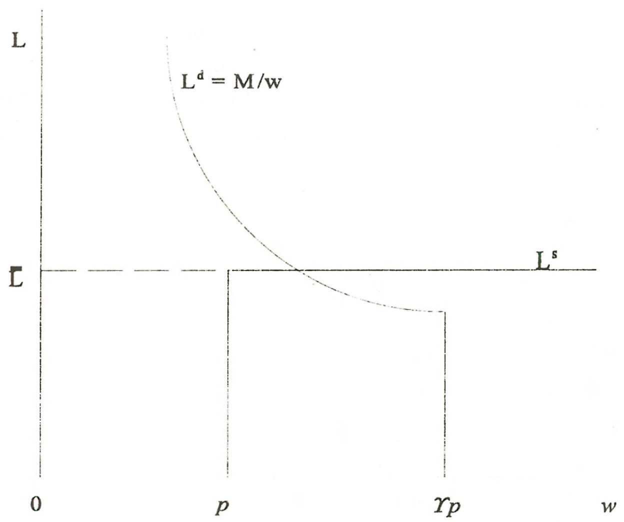
We first eliminate the equality constraints by substituting for $c_{1,t}$, q^d_{1t} and $c_{2,t}$, q^s_{1t} respectively. Now, observing that only contemporenous values of L_t appear in the objective function of both types at all times, we obtain labor demand and supply as a function of real wage as shown in Figure 1.

Agent 1, compares the real wage with his own productivity which is 1. If at any time t the real wage w_t / p_t is higher than 1, he will supply $\bar{L}$, if equal to 1 he will be indifferent between 0 and $\bar{L}$ and if less than 1, he will supply zero labor. Its labor supply function can be then written as

$$L_t^s(w_t / p_t) \begin{cases} = \bar{L} \text{ } if \text{ } w_t / p_t > 1 \\ \in [0, \bar{L}] \text{ } if \text{ } w_t / p_t = 1 \\ = 0 \text{ } otherwise. \end{cases} \tag{1}$$

Figure 1

Labor Supply and Demand Functions for a Given p



At any time t, if the real wage w_t/p_t is less than γ, agent 2 will spend all his money on purchasing labor, so the cash-in-advance (CIA) constraint will be binding. If $w_t/p_t = \gamma$, the amount of labor that he demands will be between zero and $M_{2,t}/p$. If $w_t/p_t > \gamma$, his labor demand will be zero. So his labor demand function will be

$$L_t^d(w_t / p_t) \begin{cases} = M_{2,t} \; if \; w_t / p_t > \gamma \\ \in [0, M_{2,t}] \; if \; w_t / p_t = \gamma \\ = 0 \; otherwise. \end{cases} \qquad (2)$$

The below proposition characterizes stationary monetary competitive equilibrium over the parameter space (β,γ).

Proposition. If $\beta\gamma < 1$, there exists no stationary monetary competitive equilibrium (SMCE) of the economy described in section 2. If $\beta\gamma \geq 1$, then the set of SMCE is given by (3)-(7) for each t:

$$w_t = \begin{cases} M / \bar{L} & \text{if } \beta\gamma > 1 \\ \bar{w} \in [M / \bar{L}, \infty] & \text{if } \beta\gamma = 1 \end{cases} \tag{3}$$

$$p_t = \begin{cases} M / (\beta\gamma\bar{L} & \text{if } \beta\gamma > 1 \\ \bar{w} & \text{if } \beta\gamma = 1 \end{cases} \tag{4}$$

$$L_t^d = L_t^s = \begin{cases} \bar{L} & \text{if } \beta\gamma > 1 \\ M / \bar{w} & \text{if } \beta\gamma = 1 \end{cases} \tag{5}$$

$$p_t^d = p_t^s = \begin{cases} \beta\gamma\bar{L} & \text{if } \beta\gamma > 1 \\ M / \bar{w} & \text{if } \beta\gamma = 1 \end{cases} \tag{6}$$

$$M_{1,t} = 0\,, \qquad M_{2,t} = M \tag{7}$$

Proof. We first eliminate q_t^d, c_{1t} and q_t^s, c_{2t} in (P1) and (P2) respectively since we observe that if any two of the money, labor and apple markets clear, then the third one will also clear. This follows from the third equality constraints in both (P1) and (P2).

Now, we see that labor market is equilibrium only if $w_t / p_t \in [1, \gamma]$. We have to consider then three cases:

Case 1. $1 < w_t / p_t < \gamma$.

Agent 1 will supply all the labor endowment at all periods. Moreover, since $\beta < 1$ and real return from holding money is zero, he will spend all his wage earnings in purchasing apples at each period.[2] That is, $L_t^s = \bar{L}$ and $M_{1,t} = 0$ will be the choices of agent 1 at all times.

For agent 2, since $\gamma > w_t / p_t$, the CIA constraint in labor market will be binding and we can substitute $L_t^d = M_{2,t} / w_t$ into (P2). After the substitution, the Euler equation associated with control $M_{2,t+1} / p_t$ for agent 2 is given by:

$$U'(c_{2,t}) = \beta\gamma\,(p_t / w_{t+1})\,U'(c_{2,t+1})$$

At a stationary equilibrium we have $c_{2,t+1} = c_{2,t}$ and $w_{t+1} = w_t$ for all t. Since $U'(.) > 0$ by assumption, it follows that $w_t / p_t = \beta\gamma$ must hold. So $\beta\gamma > 1$ guarantees that $1 < w_t / p_t < \gamma$ since $\beta < 1$ by assumption.

For money market clearing, we must have $M_{2,t} = M$ since $M_{1,t} = 0$ for all t. In this case consumption becomes constant over time as required by stationarity. Therefore in a SMCE, in each period, the *total* money holdings, M in the economy which is initially owned by agent 2, will be used in the first half of the period by agent 2 to buy the labor from agent 1 and then in the second half of the period by agent 1 to buy apples from agent 2, yielding a stationary equilibrium in the money market. That is, at the end of each time period the total money holdings of each agent will be equal to what he or she is initially endowed with $M_{1,t}=0$ and $M_{2,t}=M$ for all t, so that $M_{1,t} + M_{2,t} = M$ will trivially hold.

As $1 < w_t / p_t < \gamma$, we have $L_t^d = L_t^s = \bar{L}$ and therefore $w_t = M / \bar{L}$ or all t. We can then calculate $p_t = w_t / (\beta\gamma) = M / (\beta\gamma\bar{L})$ for all t, and $q_t^d = q_t^s = M / p_t = \beta\gamma\bar{L}$ for all t.

Finally, we check that the transversality condition for agent 2, that is

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[2] This is true if the prices and wages are constant over time and initial money balances are zero. Studying exercise 5.17 in Stokey and Lucas with Prescott (1989: 126-8) shows that even if initial money stock were not zero, after a finite number of periods, the consumer would choose to run it down to zero. This is the case whenever the gross real return from savings is low compared to the gross rate of time preference $1/\beta$.

$$\lim_{t\to\infty} \beta^t \frac{\gamma}{w_t} U'(c_{2t}) = 0$$

is satisfied, since both consumption and nominal wage rate is constant over time.

Euler equation together with the transversality condition above makes sure that the welfare of agent 2 is maximized as well at the described equilibrium.

The inequality constraints in both (P1) and (P2) are also satisfied. Therefore in this possible range of real wages the SMCE is unique and leads to $w_t / p_t = \beta\gamma$ for all t.

Case 2. $w_t / p_t = 1$.

Using equations (1) and (2), we note that when $w_t / p_t = 1$ the possibilities for labor traded at each period will consist of the interval $[0,\bar{L}]$, and again after substituting $L_t^d = M_{2,t} / w_t$ into (P2), we obtain the same Euler condition for agent 2 as in Case 1. At a stationary equilibrium we must have $c_{2,t+1} = c_{2,t}$ for all t, so $w_t / p_t = \beta\gamma$ is consistent with optimality whenever $w_t / p_t = 1$ if and only if $\beta\gamma = 1$.

Again due to a similar reasoning as in Case 1, the money market will be in stationary equilibrium with $M_{2,t} = M$ and $M_{1,t} = 0$ for all t. For all wages $w_t = \bar{w} \in (M/\bar{L},\infty)$ the labor market will be in stationary equilibrium with $L_t^d = L_t^s = M / \bar{w}$ for all t. The apple prices will be $p_t = \bar{w}$ for all t since $w_t / p_t = 1$ for all t. So the quantities supplied and demanded of apples will be $q_t^d = q_t^s = M / \bar{w}$ for all t. Again, one can check that transversality condition is satisfied for the second agent and that all inequality constraints in (P1) and (P2) are met.

Case 3. $w_t / p_t = \gamma$.

We will show there is no SMCE in this case. To see this, consider that when $w_t / p_t = \gamma$, the objective function of agent 2 in (P2) becomes

$$\sum_{t=0}^{\infty} \beta^t U\left(\frac{1}{p_t}(M_{2t} - M_{2,t+1})\right).$$

Under constant prices, the Euler condition for the agent 2, associated with the control $M_{2,t+1} / p_t$, is then given by

$$\beta\, U'(c_{2,t+1}) = U'(c_{2,t}).$$

At a stationary equilibrium we must have $c_{2,t+1} = c_{2,t}$ and $M_{2,t+1} = M_{2,t}$ for all t, which are impossible to hold in the above Euler equation since $\beta < 1$, contradicting that $w_t / p_t = \gamma$ supports a SMCE. **Q.E.D.**

It is interesting to observe that in the whole class of SMCE proposed by (3)-(7) the real wage rate $w/p = \beta\gamma$ is always lower than the marginal product of labor γ of the second agent's technology. So, in the equilibria producers are left with positive pure profits in contrast with an Arrow-Debreu equilibrium. The existence of this result, of course, rests upon the assumption that the contracts on apples are not allowed. In an economy where complete contracts on commodities are available, agent 2, who is endowed with money and superior technology but with zero labor, would be left with zero profits, since the perfect competition equalizes real wage rate with the marginal (and in this case the average) product of labor.

Another observation is that the higher the marginal product of labor γ or the discount rate β, the lower the equilibrium price of apples. That is in more patient or productive societies, workers would buy the apple at a lower price, and consume more of it.

A natural question to be asked is whether quantity of money is neutral in SMCE. In cases where $\beta\gamma > 1$, the answer is trivially yes. Equations (3) and (4) imply that an increase in money supply in the economy (through an helicopter drop of money to the agent 2 at time zero, for example) yields a proportional increase in all the nominal wages and prices in the economy, leaving the real prices and thus production and consumption levels in the economy unchanged. Under the case $\beta\gamma = 1$, however, the nature of the stationary monetary competitive equilibria allows for situations where money is not neutral as well as it is. That is, when the nominal wages w_t are exogenously chosen in the interval of $\bar{w} \in [M/\bar{L}, \infty)$ and fixed there, as equation (3) suggests, an increase in nominal money holdings gives rise to an equal amount of increase in labor transacted and hence quantity of apples produced, which can be seen by equations (6) and (5). But, since the economic environment that we consider in this paper is not rich enough to identify an equilibrium selection mechanism for nominal wages when $\beta\gamma=1$, one may equally be justified in claiming that nominal wages and prices always move together with money holdings, leaving the real side of the economy unchanged. So, we face a situation of indeterminacy, involving an extreme yet an interesting possibility for non-neutrality of money in the context of a model with optimizing agents, perfect foresight and market clearing.

4. Conclusions

In this paper, we examined a very simple model with two types of representative agents, one owning an inferior technology and all the labor endowment and the other owning a superior technology and all the initial money available. We showed that under certain conditions there exists stationary monetary competitive equilibria under which workers receive less than their marginal products and competition cannot sweep out pure profits even under constant returns to scale technologies. This striking observation is simply because enterpreneurs face cash-in-advance constraints in the factor markets.

We also showed that under a very specific case, multiple equilibria may arise in the economy, allowing for situations where money is not neutral.

Although we only dealt with identifying the class of SMCE, it may be of an interest to characterize non-stationary MCE as well.

The results would be substantially different if we were to give some initial money to consumer-workers and let the good market open first and then let the labor market open. So sequencing of markets seems to be crucial. However we would like to argue that the sequencing we study here is more realistic since purchases of goods before production takes place (forward commodity contracts) is not so commonplace. But since we do not allow inventory accumulation, such forward contracts are necessary if the good market is to be opened first.

A very fruitful extension of this study would be to allow for the presence of a credit market where an entrepreneur could borrow cash to finance its working capital needs before the labor market opens. The repayment would naturally take place after the closing of the goods market in that case. A finite horizon version of such a model was studied in Başçı (1995), where money is backed by the government. Naturally in such a case if consumer-workers are endowed with initial